ON THE USE OF AN ASYMPTOTIC METHOD FOR ANALYZING WAVES WITH UNSTEADY MOTION OF THE SOURCE

(O PRIMENENII ASIMPTOTICHESKOGO METODA DLIA ANALIZA VOLN PRI NEUSTANOVIVSHEMSIA DVIZHENII ISTOCHNIKA)

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A problem involving unsteady rectilinear motion of a source in a fluid of finite depth is considered. A similar problem for a pressure pulse applied to the surface of a deep fluid was considered in [1] with the aid of the integral transform method.

Extensive use in the solution of wave problems has lately been made of Green's functions constructed in various ways. This approach is used here to investigate the unsteady motion of hydrodynamic singularities.

1. Following Kelvin, let us consider the unsteady motion of a source of intensity $Q(t_1)$ as the result of the superimposition of disturbances from several pulse sources that exist for infinitesimal time intervals Δt_1 and at each point of the path traversed by the source displace a volume of fluid $Q(t_1)\Delta t_1$. We then obtain the following expression for a source situated at the point $(0, 0, z_1)$ of the bound coordinate system (*) moving with the velocity $c(t_1)$ in the positive direction along the x-axis

$$\Phi = \int_0^t Q(t_1)\,\varphi(x+s, y, z, z_1, t-t_1)\,dt_1, \qquad s = \int_{t_1}^t c(\tau)\,d\tau \qquad (1.1)$$

where $\varphi(x, y, z, z_1, t-t_1)$ is a potential of a pulse source of unit intensity.

To determine the potential φ we can make use of the time-dependent Green's function [2]

$$G = \frac{\theta(t-t_1)}{2\pi}\int_0^\infty \frac{\cosh k(z_1+H)}{\cosh kH}\left[\sinh kz - \frac{\cosh k(z+H)}{\cosh kH}\,\frac{1-\cos\sigma_1(t-t_1)}{\tanh kH}\right] J_0(kr)\,dk$$

$$r = \sqrt{x^2+y^2}, \qquad \sigma_1 = \sqrt{gk\tanh kH} \qquad (1.2)$$

*) With the origin lying on a free surface and the z-axis directed vertically upward.

The function G gives the solution of the problem of a source of unit intensity which arises at the instant t_1 at some point of the stationary fluid and then continues to exist unchanged.

Since it is important in the discussion to follow that no motion occurs for $t < t_1$, the right-hand side of (1.2) has been multiplied by the function $\theta(t - t_1)$ which is equal to one for $t > t_1$ and to zero for $t < t_1$.

In order to obtain the potential of the pulse source it is clearly sufficient to consider the difference in the functions G for the two sources of intensity q whose time of appearance differs by an infinitely small amount Δt_1, and to have $\Delta t_1 \to 0$. If, further, we stipulate that $q\Delta t_1 \to 1$, we find that

$$\varphi = q\,[G(t-t_1) - G(t-t_1-\Delta t_1)]|_{\Delta t_1\to 0} = -q\frac{\partial G}{\partial t_1}\Delta t_1 = \frac{\partial G}{\partial t} \tag{1.3}$$

or, differentiating (1.2), that

$$\varphi = \frac{\delta(t-t_1)}{2\pi}\int_0^\infty \frac{\sinh kz \cosh k(z_1+H)}{\cosh kH} J_0(kr)\,dk -$$

$$-\frac{\theta(t-t_1)\,g}{2\pi}\int_0^\infty \frac{k\cosh k(z+H)\cosh(z_1+H)}{\sigma_1 \cosh^2 kH}\sin\sigma_1(t-t_1)\,J_0(kr)\,dk \tag{1.4}$$

where $\delta(t - t_1)$ is the Dirac delta function.

To obtain the potential of the moving source in accordance with (1.1), we integrate (1.4),

$$\Phi = \frac{Q(t)\,\theta(t)}{2\pi}\int_0^\infty \frac{\sinh kz \cosh k(z_1+H)}{\cosh kH} J_0(kr)\,dk -$$

$$-\frac{g}{2\pi}\int_0^t Q(t_1)\,dt_1 \int_0^\infty \frac{k\cosh k(z+H)\cosh k(z_1+H)}{\sigma_1 \cosh kH}\times$$

$$\times \sin\sigma_1(t-t_1)\,J_0[k\sqrt{(x+s)^2+y^2}]\,dk \tag{1.5}$$

whence as $H \to \infty$ we have the familiar solution of Sretenskii [3].

The ordinates of the free surface under the usual assumptions of small-amplitude wave theory can be defined as follows:

$$\zeta^\circ = \frac{1}{g}\left[c\frac{\partial\Phi}{\partial x} - \frac{\partial\Phi}{\partial t}\right]\Big|_{z=0} \tag{1.6}$$

In the simplest case where a source of intensity Q = const begins to move from the state of rest with a constant velocity c, i.e. where $s = (t - t_1)c$, we have (1.7)

$$\zeta = \zeta^\circ : \frac{Qc}{2\pi g H^2} = \frac{1}{v}\int_0^\infty \frac{\alpha\cosh\alpha(1+\zeta_1)}{\cosh\alpha}\int_0^\tau J_0[\alpha\sqrt{(\xi+v\tau_1)^2+\eta^2}]\cos\sigma\tau_1\,d\tau_1\,d\alpha$$

In this expression

$$\xi = \frac{x}{H}, \qquad \eta = \frac{y}{H}, \qquad \zeta_1 = \frac{z_1}{H}, \qquad v = \frac{c}{\sqrt{gH}}$$

$$\alpha = kH, \qquad \sigma = \sqrt{\alpha \tanh \alpha}, \qquad \tau = t\left(\frac{g}{H}\right)^{1/2}, \qquad \tau_1 = (t - t_1)\left(\frac{g}{H}\right)^{1/2}$$

Changes in the ordinates of the free surface $\zeta = \zeta(\tau)$ at various instants of time are shown in Fig.1, which contains the results of computing integral (1.7) for $\eta = 0$, $v = 0.5$ and $\zeta_1 = -0.5$ on a computer (curve 1).

As $\tau \to \infty$, (1.7) can be used to obtain expressions for the ordinates of the free surface with steady source motion. Specifically, for $\eta = 0$ we have

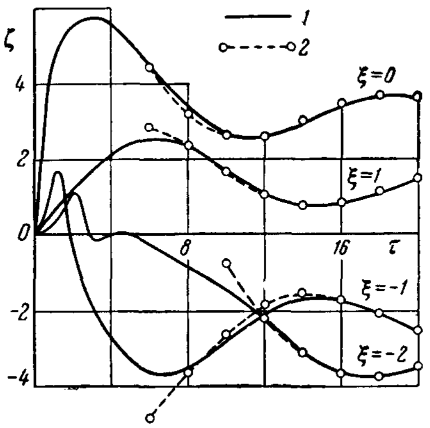

Fig. 1

$$\zeta^{(\infty)} = S_1 + S_2 - S_3 \qquad (1.8)$$

$$S_1 = \frac{1}{v}\int_0^{\alpha_0} \frac{\alpha \cosh \alpha\,(1+\zeta_1)}{\cosh \alpha} \sin\frac{\sigma\xi}{v} \frac{d\alpha}{\sqrt{\sigma^2 - \alpha^2 v^2}}$$

$$S_2 = \frac{1}{v}\int_{\alpha_0}^{\infty} \frac{\alpha \cosh \alpha\,(1+\zeta_1)}{\cosh \alpha} \cos\frac{\sigma\xi}{v} \frac{d\alpha}{\sqrt{\alpha^2 v^2 - \sigma^2}}$$

$$S_3 = \frac{1}{v^2}\int_0^{\infty} \frac{\alpha \cosh \alpha\,(1+\zeta_1)}{\cosh \alpha} \int_0^{\xi} J_0(\alpha\beta) \times$$

$$\times \cos\frac{\sigma(\xi-\beta)}{v}\, d\beta\, d\alpha$$

where α_0 is a root of the equation $\sigma = \alpha v$.

Fig. 2 (curve 1) depicts the shape of the free surface with steady motion of the source just considered. Trial calculations show that for large values of ξ and τ, numerical integration of (1.7) and (1.8) becomes practically impossible due to the inordinately large amount of computer time required.

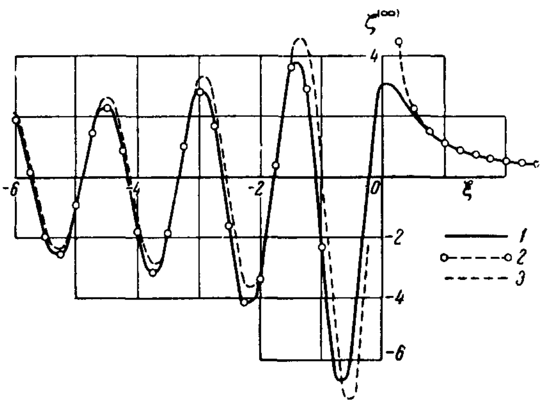

Fig. 2

These difficulties have to do with the oscillatory character of the integrands in (1.7) and (1.8). At the same time, we know that integrals of rapidly oscillating functions are approximated quite closely by their asymptotic expansions. Such expansions can be obtained, for example, with the aid of the stationary phase principle. But the stationary phase formula gives only the first term of the asymtotic expansion and can be applied only when the denominator of the integrand does not vanish. For this reason, we will obtain the asymptotic expansions of integrals of more general form.

2. The integrals of rapidly oscillating functions encountered in the solution of wave problems can in most cases be written in the form

$$I = \int_{\alpha}^{\gamma} \frac{f_1(u)}{f_2(u)} e^{ik\varphi(u)}\, du \tag{2.1}$$

where k is a large parameter, and $\varphi(u)$, $f_1(u)$ and $f_2(u)$ are differentiable for $\alpha \leqslant u \leqslant \gamma$.

If $\varphi'(u) \neq 0$ and $f_2(u) \neq 0$ within the limits of integration, then in order to obtain the asymptotic expansion it is sufficient to integrate (2.1) by parts, whereupon each integration yields the next succeeding term of the expansion [4].

Cases involving "singularities" – zeros in the denominator or zero values of the derivative of the phase function in the trigonometric factor (stationary points) require special consideration. It turns out to be possible to represent the integral under investigation as a sum of two terms, of which one has the same singularity as the integral being considered but is integrable in closed form, while the other contains no singularities and is integrable by parts.

For $\alpha < \beta < \gamma$ let us have $\varphi(\beta) = 0$ and $f_2(\beta) = 0$, and $\varphi'(\beta) \neq 0$ and $f_2'(\beta) \neq 0$. Introducing the notation $\psi(u) = \varphi(u) f_1(u) / f_2(u)$, where $\psi(u)$ is differentiable, we then obtain

$$I = \int_{\alpha}^{\gamma} \frac{\psi(u)}{\varphi(u)} e^{ik\varphi(u)}\, du = \int_{\alpha}^{\gamma} \left[\frac{\psi(u)}{\varphi'(u)} - \frac{\psi(\beta)}{\varphi'(\beta)}\right] \frac{e^{ik\varphi(u)}}{\varphi(u)}\, \varphi'(u)\, du + \frac{\psi(\beta)}{\varphi'(\beta)} \int_{\varphi(\alpha)}^{\varphi(\gamma)} \frac{e^{ikw}}{w}\, dw \tag{2.2}$$

The first term on the right-hand side of (2.2) contains no singularities and can be integrated by parts; the second may be written as

$$\int_{\varphi(\alpha)}^{\varphi(\gamma)} \frac{1}{w} e^{ikw}\, dw = -\int_{-\infty}^{\varphi(\alpha)} \frac{1}{w} e^{ikw}\, dw - \int_{\varphi(\gamma)}^{\infty} \frac{1}{w} e^{ikw}\, dw + \int_{-\infty}^{\infty} \frac{1}{w} e^{ikw}\, dw \tag{2.3}$$

If $k > 0$ and $\varphi(\alpha) < \varphi(\gamma)$, i.e. $k\varphi'(\beta) > 0$, the first and second terms in the right-hand side of (2.3) can also be integrated by parts, and the last one can be computed in closed form. Finally, since $k\varphi'(\beta)$ can be less than zero, we find that

$$\int_{\alpha}^{\gamma} \frac{\psi(u)}{\varphi(u)} e^{ik\varphi(u)}\, du = \frac{\pi i \psi(\beta)}{\varphi'(\beta)} \operatorname{sign} k\varphi'(\beta) + C(u)\Big|_{u=\alpha}^{u=\gamma} \tag{2.4}$$

where $C(u)$ is obtained by formal integration by parts of the left-hand side of (2.4).

Asymptotic expansions for other types of integrand singularities can be obtained in a similar way.

In particular, for $\varphi'(\beta) = 0$ and $\varphi''(\beta) \neq 0$ we have

$$\int_{\alpha}^{\gamma} \psi(u)\, e^{ik\varphi(u)}\, du = \psi_0 \left(\frac{\pi}{|k\varphi_2|}\right)^{1/2} \left\{1 - \frac{1}{2ik}\left[\frac{15\varphi_3^2 - 12\varphi_2\varphi_4}{8\varphi_2^3} - \frac{3\psi_1\varphi_3}{2\psi_0\varphi_2^2} + \right.\right.$$

$$\left.\left. + \frac{\psi_2}{\psi_0\varphi_2}\right] + O\left(\frac{1}{k^2}\right)\right\} \exp\left(ik\varphi_0 + \frac{\pi i}{4}\,\mathrm{sign}\, k\varphi_2\right) + C(u)\Big|_{u=\alpha}^{u=\gamma} \qquad (2.5)$$

$$\psi_n = \frac{1}{n!}\frac{\partial^n \psi}{\partial u^n}\Big|_{u=\beta}, \qquad \varphi_n = \frac{1}{n!}\frac{\partial^n \varphi}{\partial u^n}\Big|_{u=\beta}$$

The first term of this expansion gives the formula for the stationary phase.

The formulas just obtained imply that the asympotic expansion of the integral in the presence of singularities consists of two parts, of which one is obtained by formal integration by parts, while the other is associated with the presence of a singularity and is determined by the nature of that singularity. It is also clear that if several singular points occur within the limits of integration, their individual contributions must be added together.

3. To apply the asymptotic formulas obtained above we replace the Bessel function in (1.7) by its integral representation

$$J_0(\alpha\sqrt{\xi^2+\eta^2}) = \frac{1}{\pi}\int_0^{\pi} \cos\,[\alpha(\xi\cos\theta + \eta\sin\theta)]\, d\theta$$

and integrate over the time

$$\zeta = \frac{1}{2\pi v}\sum_{n=1}^{2}\int_0^{\pi} d\theta \int_0^{\infty} \frac{\alpha\cosh\alpha\,(1+\zeta_1)}{\cosh\alpha}\cdot\left\{\frac{(-1)^{n+1}\sin\,[\alpha\rho\cos(\vartheta-\theta)]}{\sigma + (-1)^n\alpha v\cos\theta} + \right.$$

$$\left. + \frac{\sin\,[\sigma\tau + (-1)^n\alpha v\tau\cos\theta + (-1)^n\alpha\rho\cos(\vartheta-\theta)]}{\sigma + (-1)^n\alpha v\cos\theta}\right\} d\alpha \qquad (3.1)$$

$$(\rho = \sqrt{\xi^2+\eta^2}, \quad \vartheta = \tan^{-1}(\eta/\xi), \quad \eta > 0)$$

We confine ourselves to the case of subcritical velocities ($v < 1$). For $n = 1$ the denominators of the inner integral can vanish; in addition, the second term can have stationary points.

For large values of ρ, when the order of magnitude of the ratio τ/ρ is equal to unity, the contribution due to the first of the aforementioned singularities can be obtained with the aid of Formula (2.4)

$$\zeta^{(1)} = \frac{1}{2v}\int_0^{\pi} \frac{\alpha_i^\circ\cosh\alpha_i^\circ\,(1+\zeta_1)\cos\,[\alpha_i^\circ\rho\cos(\vartheta-\theta)]}{\cosh\alpha_i^\circ\,(\sigma' - v\cos\theta)}\left\{\mathrm{sign}\,[\cos(\vartheta-\theta)] + \right.$$

$$\left. + \,\mathrm{sign}\left[\sigma' - \frac{\sigma}{\alpha_i^\circ} - \frac{\rho}{\tau}\cos(\vartheta-\theta\right]\right\} d\theta \qquad (3.2)$$

$$\left(\sigma = \sigma(\alpha_i^\circ),\ \sigma' = \frac{d\sigma}{d\alpha} \ \text{ for } \ \alpha = \alpha_i^\circ\right)$$

where α_i° is a root of Equation

$$\sigma = \alpha v\cos\theta \qquad (3.3)$$

The expression within the braces in the right-hand side of (3.2) differs from zero for

$$|\vartheta-\theta|>\frac{\pi}{2}, \qquad \frac{\sigma}{\alpha_i^{\circ}}-\sigma'>-\frac{\rho}{\tau}\cos(\vartheta-\theta) \tag{3.4}$$

In this case the integral over θ has stationary points. Differentiating the phase function of the trigonometric factor and recalling that θ and α_i° are related by expression (3.3), we obtain the following condition for determining the value of α_i corresponding to the stationary points

$$\frac{\partial}{\partial\theta}[\alpha_i^{\circ}\cos(\vartheta-\theta)]=\frac{\alpha_i^{\circ 2}v\sin\theta}{\sigma-\alpha_i^{\circ}\sigma'}\cos(\vartheta-\theta)+\alpha_i^{\circ}\sin(\vartheta-\theta)=0 \tag{3.5}$$

This equation can have two roots (i = 1, 2), so that with the aid of Formula (2.5) we obtain

$$\zeta^{(1)}=\frac{\sqrt{2\pi}}{v}\sum_{i=1}^{2}\frac{\alpha_i\cosh\alpha_i\,(1+\zeta_1)\,[\alpha_i^2(v^2+\sigma'^2)-2\alpha_i\sigma\sigma']^{1/4}}{(\xi^2+\eta^2)^{1/4}\cosh\alpha_i\,[\alpha_i\sigma''(\alpha_i^2v^2-\sigma^2)+\sigma'(\sigma-\alpha_i\sigma')^2]^{1/2}}\times$$
$$\times\cos\left[\frac{\sigma|\xi|}{v}-\frac{\sqrt{\alpha_i^2v^2-\sigma^2}}{v}\eta-(-1)^i\frac{\pi}{4}\right] \tag{3.6}$$

where σ, σ' and σ'' should be taken for $\alpha=\alpha_i$.

Let us consider the existence conditions for the roots of Equation (3.5). To this end, we use (3.3) to eliminate θ,

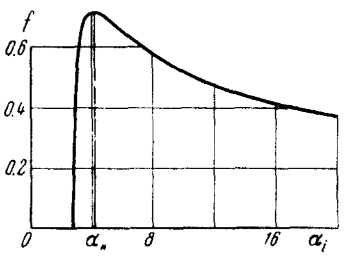

Fig. 3

$$\tan\vartheta=-f(\alpha_i),\; f(\alpha_i)=\frac{\sigma'\sqrt{\alpha_i^2v^2-\sigma^2}}{\alpha_iv^2-\sigma\sigma'} \tag{3.7}$$

Fig.3 shows the graph of the function $f(\alpha_i)$ for v = 0.6 , whence it follows that (3.7) can have roots only for points lying within the angle

$$\pi>\vartheta>\tan^{-1}[-f(\alpha_*)] \tag{3.8}$$

The other boundary is given by condition (3.4), which upon elimination of θ becomes

$$\frac{\rho}{\tau}<\left((v^2+\sigma'^2)-\frac{2\sigma\sigma'}{\alpha_i}\right)^{1/2}$$

or

$$0>\frac{\xi}{\tau}>-\frac{\alpha_iv^2-\sigma\sigma'}{\alpha_iv}, \qquad \frac{\eta}{\tau}<\frac{\sigma'\sqrt{\alpha_i^2v^2-\sigma^2}}{\alpha_iv} \tag{3.9}$$

Fig. 4 gives an overall view of the boundaries obtained above for v = 0.6 and τ = 10.5 as well as the shapes of the equal-phase lines which are obtained by equating the argument of the cosine in the right-hand side of (3.6) to $m\pi$ (m = 1, 2,...). This condition together with (3.7) yields a system of two equations with the parameter α_i whose value may be set arbitrarily. The geometric locus of the points thus obtained (broken lines in Fig.4) corresponds to the locus of the crests and troughs of the wave system in the wake of the source. Analysis of the shape of these curves reveals that the points corresponding to $\alpha_i<\alpha_*$ (i = 1) give a system of transverse waves, and those for which $\alpha_i>\alpha_*$ (i = 2) are associated with divergent waves.

Condition (3.9) implies that there is a region in the wake of the source where only divergent waves are present (*).

As $H \to \infty$ Formula (3.8) becomes the usual condition for the wake boundary $\pi > \vartheta > \tan^{-1}(-1/\sqrt{8})$, and (3.9) yields the circle

$$\left(\frac{\xi}{\tau}+\frac{3v}{4}\right)^2+\left(\frac{\eta}{\tau}\right)^2=\left(\frac{v}{4}\right)^2 \qquad (3.10)$$

In addition to the singularities considered above, the inner integral in (3.1) has stationary points for $\alpha = \alpha_k{}^\circ$ that satisfy Equation

$$\frac{\partial\varphi_1}{\partial\alpha}=\sigma'+(-1)^n v\cos\theta+(-1)^n\frac{\rho}{\tau}\cos(\vartheta-\theta)=0 \qquad (3.11)$$

Then, applying Formula (2.5), for large τ we have

$$\zeta^{(2)}=\frac{1}{v\sqrt{2\pi}}\int_0^\pi\frac{\alpha_k{}^\circ\cosh\alpha_k{}^\circ(1+\zeta_1)}{\sqrt{\tau|\sigma''|}\cosh\alpha_k{}^\circ}\,\frac{\sin[(\sigma-\alpha_k{}^\circ\sigma')\tau-{}^1/_4\pi]}{\sigma+(-1)^n\alpha_k{}^\circ v\cos\theta}\,d\theta \qquad (3.12)$$

Integral (3.12) can also have stationary points given by condition

$$\frac{\partial\varphi_2}{\partial\theta}=(-1)^{n+1}\alpha_k{}^\circ\left[v\sin\theta-\frac{\rho}{\tau}\sin(\vartheta-\theta)\right]=0 \qquad (3.13)$$

or, upon elimination of θ with the aid of (3.11), by condition

$$(v+\xi/\tau)^2+(\eta/\tau)^2=\sigma'^2 \qquad (3.14)$$

It can be shown that Equation (3.11) for $\eta > 0$ has solutions only for $n = 1$; in this case, in accordance with Formula (2.5), we obtain

$$\zeta^{(2)}=\frac{1}{v\tau}\frac{\cosh\alpha_k(1+\zeta_1)}{\cosh\alpha_k}\left(\frac{\alpha_k}{\sigma'|\sigma''|}\right)^{1/2}\frac{\sin(\sigma-\alpha_k\sigma')\tau}{\sigma-\alpha_k v\cos\theta_k}$$

$$\cos\theta_k=\frac{v^2+\sigma'^2-(\rho/\tau)^2}{2v\sigma'} \qquad (3.15)$$

where α_k is a root of Equation (3.13) or (3.14).

Condition (3.14) implies that stationary points are possible only for

$$(v+\xi/\tau)^2+(\eta/\tau)^2<1 \qquad (3.16)$$

Beyond this region $\zeta^{(2)}=0$.

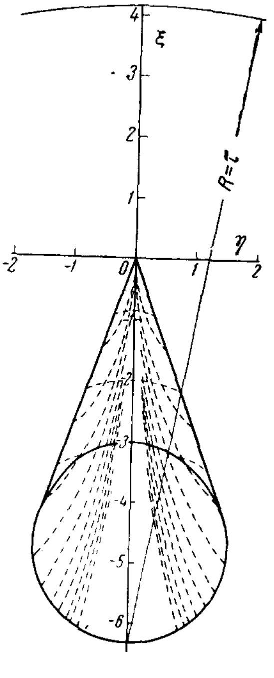

Fig 4

As has already been pointed out, in order to obtain the asymptotic expansion sought, we must add to (3.6) and (3.15) the result of integrating by parts the inner integral of

*) [1] contains the erroneous statement that only transverse waves are present in this region (p. 729). As is shown below, this conclusion conflicts with the laws of energy transfer and wave propagation in fluids.

(3.1), which gives

$$\zeta^{(0)} = \frac{1}{v\tau} \frac{(v\tau + \xi)\xi + \eta^2}{\xi^2 + (1 - v)\eta^2} \left[1 - \left(v + \frac{\xi}{\tau}\right)^2 - \left(\frac{\eta}{\tau}\right)^2\right]^{-1/2} + O\left(\frac{1}{\tau^2}\right)$$

This component likewise differs from zero only within the circle (3.16).

We finally obtain

$$\zeta = \zeta^{(0)} + \zeta^{(1)} + \zeta^{(2)} \tag{3.17}$$

As $\tau \to \infty$ this clearly implies the asymptotic expansion for the ordinates of the free surface with steady source motion. Fig.2 (curve 2) show the results of asymptotic computation for $\eta = 0$ and $\tau \to \infty$ with the term of order ~ in Formula (3.6) taken into account. The resulting values are already in good agreement with the exact results for $\xi \approx 1$. For comparison, the figure also includes results calculated with the aid of the stationary phase formula (curve 3).

Similar calculations were also carried out for the case of unsteady motion. As is clear from the derivation, Formula (3.15) must be valid for all ρ including small values; this applies to $\zeta^{(0)}$, as well provided that its time-independent component is isolated. Hence, replacing the asymptotic expansion of the steady-motion wave ordinates by their exact values in (3.17), we obtain an asymptotic formula applicable for all ρ

$$\zeta = \zeta^{(\infty)} + \zeta^{(2)} + \zeta^{(0)} - \frac{\xi}{\xi^2 + (1 - v)\eta^2} \frac{1}{\sqrt{1 - v^2}} \tag{3.18}$$

The results of computations using Formula (3.18) shown in Fig.1 (curve 2) indicate that this asymptotic formula is highly accurate in that range of values of τ where computation by numerical integration becomes difficult.

A similar method can be employed to investigate the case of disappearance of a source hitherto moving with a constant velocity which in the first approximation describes the behavior of waves during retardation of the source. All that is required is to find the difference in the ordinates of the waves during steady motion (1.8) and during acceleration (1.7).

It should be noted in conclusion that the ranges of existence of individual asymptotic expansion components found above are in full agreement with the laws of propagation of disturbances in a fluid of finite depth. Indeed, their velocity cannot exceed the critical value, so that the fluid remains at rest outside the circle (3.16). Further, for transverse waves which turn out to be practically flat near the line $\eta = 0$ (Fig.4), the group velocity is equal to σ' in accordance with (3.2) and (3.7). The point which starts from the origin of the coordinate system bound to the source and moves with a velocity σ' along the x-axis for $\tau > 0$ reaches the uppermost point of region (3.10) by the instant τ in question. For this reason, all of the energy of the transverse waves, which, as we know, is propagated with the group velocity [5], turns out to have been carried over the upper boundary of region (3.10) by the instant τ , so that transverse waves are fully decayed with this region.

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